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Phenomenology of electroweak symmetry breaking from theory space

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Abstract: Recently, a new class of realistic models for electroweak symmetry breaking have been constructed, without supersymmetry. These theories have naturally light Higgs bosons and perturbative new physics at the TeV scale. We describe these models in detail, and show that electroweak symmetry breaking can be triggered by a large top quark Yukawa coupling. A rich spectrum of particles is predicted, with a pair of light Higgs doublets accompanied by new light weak triplet and singlet scalars. The lightest of these new scalars is charged under a geometric discrete symmetry and is therefore stable, providing a new candidate for WIMP dark matter. At TeV energies, a plethora of new heavy scalars, gauge bosons and fermions are revealed, with distinctive quantum numbers and decay modes.

Keywords: Spontaneous Symmetry Breaking, Beyond Standard Model, Higgs Physics, Chiral lagrangians.

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1. Introduction

In this decade, experiments will begin to thoroughly explore the origin of electroweak symmetry breaking. The description of electroweak symmetry breaking in the standard model, in terms of a fundamental scalar Higgs field, is almost certainly incomplete. The quadratically divergent radiative corrections to the Higgs mass suggest the existence of new physics at TeV energies that stabilizes the weak scale. Until recently, all theories for this stabilization could be grouped into two categories: those that rely on new strong dynamics or compositeness near a TeV (such as technicolor, composite Higgs, or theories with a low fundamental Planck scale), and those with low-scale supersymmetry. Theories with low scale supersymmetry are particularly attractive since they allow a perturbative description of the physics that softens the quadratic divergences, and naturally lead to light Higgs bosons, as seems favored by precision electroweak data.

Recently, a qualitatively new category of realistic theories of electroweak symmetry breaking has been introduced [1]. These models, based on the physics of "theory space" [2] (see also [3]), offer a new mechanism for softening the quadratic divergences in the Higgs mass. Electroweak symmetry breaking is accomplished with naturally light Higgs bosons, which descend from non-linear sigma model fields whose mass is protected by "chiral" symmetries of the sigma model (The first attempts at models of this kind were the "composite Higgs" theories [4, 5]). The physics is perturbative at energies parametrically above the

TeV scale, ultimately requiring UV completion near $\sim 10 \to 100$ TeV where the non-linear sigma model fields become strongly coupled. However, the physics of electroweak symmetry breaking and the new physics at the TeV scale is weakly coupled. These models are fully realistic, incorporating fermion masses without producing dangerous flavor-changing neutral currents. Additionally, there are new stable weakly interacting scalars which can provide WIMP dark matter.

In [1] the construction of naturally light scalars with gauge, Yukawa and quartic self-couplings from theory space was described, and one fairly simple model of electroweak symmetry breaking was presented. In this paper, we describe this model in more detail and begin an exploration of its phenomenology.

We start by describing the bosonic field content and symmetries of these models which are associated with an $N \times N$ toroidal theory space (or "moose" diagram) and discuss how the light (or "little") Higgs fields emerge at low energies. Subsequently we incorporate the standard model fermions along with their Yukawa couplings. The chiral symmetry structure of the theory guarantees the absence of quadratic divergences in the little Higgs masses at low orders in perturbation theory. We then explicitly compute the leading one-loop radiative corrections to the little Higgs potential, verifying the absence of quadratic divergences. We find a calculable negative contribution to the little Higgs mass squared from the large top Yukawa coupling, which can naturally drive electroweak symmetry breaking (EWSB). Finally, we discuss the $N = 2$ model in detail, presenting the spectrum of the theory and the principal decay modes of the new states.

1.1 Fields and gauge symmetries

At energies beneath the scale $\Lambda \sim 10$–100 TeV, our theory is well-described by a gauged non-linear sigma model. The gauge sector includes $N \times N$ gauge groups G_a, where $a = (m, n)$ for $-N/2 < m, n \le N/2$ and we periodically identify $(m, n) = (m + N, n) = (m, n + N)$. We take $G_a = \mathrm{SU}(3)$ except for $G_{\mathbf{0}} = \mathrm{SU}(2) \times \mathrm{U}(1)$, where $\mathbf{0} = (0, 0)$. The non-linear sigma model fields are 3×3 unitary matrices transforming as bi-fundamentals under nearest neighbor gauge groups. The gauge group and field content of this model can be represented by a theory space, shown in figure 1 for the case $N = 4$. Each field can be labeled by a "link" $l = (a, b)$, where a and b are adjacent sites:

$$\Sigma_l = e^{i\pi_l}\,, \qquad \text{with } \Sigma_{\bar{l}} = \Sigma_l^\dagger\,. \tag{1.1}$$

We adopt the convention $\overline{(a, b)} \equiv (b, a)$. Under the gauge symmetries Σ_l corresponding to link $l = (a, b)$ transforms as

$$\Sigma_l \longrightarrow g_a \Sigma_l g_b^{-1}\,. \tag{1.2}$$

Here, $g_{\mathbf{0}} = h_{\mathrm{SU}(2)} \exp(i\frac{\theta}{\sqrt{3}}T_8)$ is an element of the gauge group $G_{\mathbf{0}}$, where $h_{\mathrm{SU}(2)}$ commutes with T_8. The kinetic part of the action, including gauge interactions, is

$$\mathcal{L}_K = -\sum_a \frac{1}{2g_a^2} trF_a^2 + \sum_l \frac{f_l^2}{4}\,\mathrm{tr}|D^\mu\Sigma_l|^2 + \cdots\,, \tag{1.3}$$

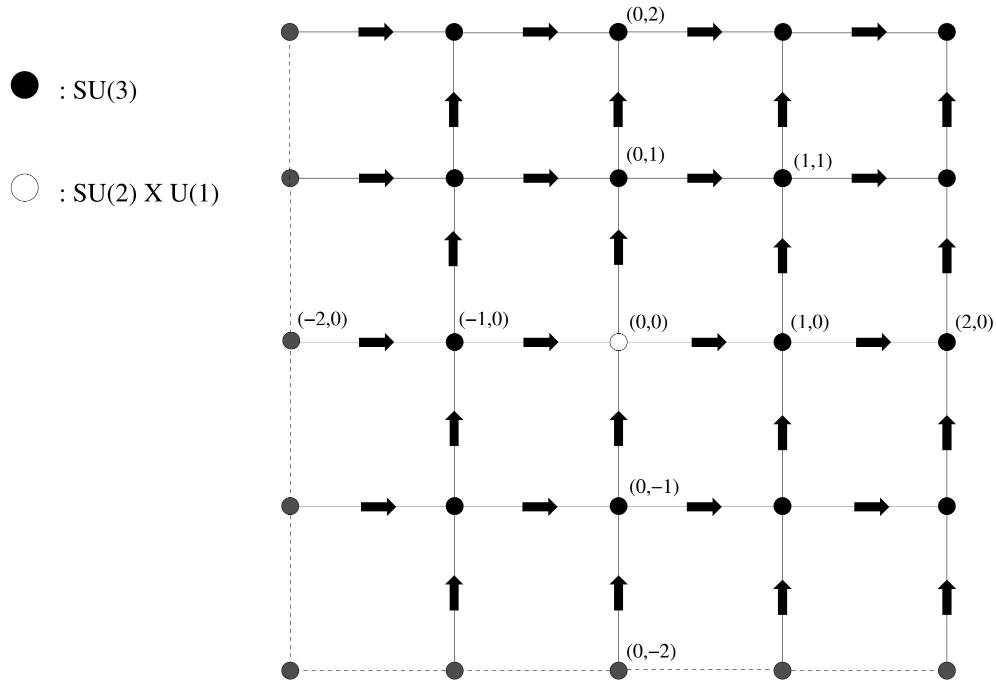

Figure 1: Theory space for the 4×4 torus. Sites $(-2, m)$ and $(2, m)$ are identified as are $(n, -2)$ and $(n, 2)$. The thick arrows represent the link fields.

with a covariant derivative

$$D^{\mu}\Sigma_l = \partial^{\mu}\Sigma_l + iA_a^{\mu}\Sigma_l - i\Sigma_l A_b^{\mu} \,. \tag{1.4}$$

For simplicity, we take all the in principle independent f_l constants to be the same. The ellipses in eq. (1.3) represent operators involving more than two derivatives suppressed by powers of the scale Λ. The natural size for this cutoff is $\Lambda = 4\pi f$, where the theory becomes strongly coupled. Since we will only be interested in energies well below Λ we will ignore these higher derivative operators. We also have the usual QCD $SU(3)_{\rm color}$ gauge symmetry, under which all these fields are neutral.

In addition to the operators in (1.3) we can include non-derivative terms as well. We must first classify all operators and the natural size of their coefficients [6, 7]. Non-derivative gauge invariant operators can be constructed from "Wilson" lines. A line ℓ is a collection of sequential links $l_1, \ldots, l_n$, where $l_i = (a_i, a_{i+1})$. The associated Wilson line W_ℓ is the product of the link fields $W_\ell = \Sigma_{l_1}\Sigma_{l_2}\cdots\Sigma_{l_n}$. Wilson loops are Wilson lines for closed paths. We will represent Wilson loops by pictures of the path with the initial site a labeled, traversed in the counterclockwise direction. Especially important will be the fundamental "plaquettes", the Wilson loops associated with the smallest square paths. For example, the symbol ${}_a\Box$ represents the operator

$${}_a\Box = \Sigma_{(a,a+\mathbf{u})}\Sigma_{(a+\mathbf{u},a+\mathbf{u}+\mathbf{v})}\Sigma_{(a+\mathbf{u}+\mathbf{v},a+\mathbf{v})}\Sigma_{(a+\mathbf{v},a)} \,, \tag{1.5}$$

where $\mathbf{u}, \mathbf{v}$ are the unit vectors on the moose $\mathbf{u} = (1,0)$, $\mathbf{v} = (0,1)$.

In classifying the natural sizes for the coefficients of operators constructed from Wilson loops we need to understand their transformation properties under the chiral symmetries carried by each link:

$$\Sigma_l \longrightarrow L_l \Sigma_l R_l^\dagger\,. \tag{1.6}$$

A Wilson line breaks the independent $R_{l_i}, L_{l_{i+1}}$ symmetry transformation to the diagonal $R_{l_i} = L_{l_{i+1}}$, for each l_i in the line. Note that the gauge couplings in (1.3) also break these chiral symmetries, and we expect that gauge interactions will then induce Wilson loop operators. But these gauge interactions must come in pairs: any breaking associated with the link Σ_ℓ must be accompanied by the conjugate breaking of $\Sigma_\ell^\dagger$. So the only operators which are induced by the gauge interactions are products of

$$trW_\ell trW_\ell^\dagger\,. \tag{1.7}$$

Since each chiral symmetry breaking is accompanied by a gauge coupling, the natural size for the coefficient of the operator in (1.7) is $16\pi^2 f^4 (g^2/(16\pi^2))^n$ where n is the number of links in ℓ. Therefore our action includes these terms, with a naturally small coefficient of this size. This power counting argument shows that this operator will be renormalized with quadratic divergences appearing only at n loops. For $N > 2$, one loop infrared contributions of order $g^4/16\pi^2$ will dominate over the ultraviolet contributions.

We can in addition add interactions that are not generated in this way, by including operators that break the chiral symmetries in a different pattern than the breaking by the gauge couplings. The coefficients of these operators are therefore in principle unrelated to the gauge couplings. We choose the largest new symmetry breaking interactions to correspond to plaquette operators $tr\,{}_a\Box$ with coefficients λ_a:

$$\mathcal{L}_{\mathrm{pl}} = f^4 \sum_a tr\,{}_a\Box\,\lambda_a + \text{h.c.}\,. \tag{1.8}$$

Other gauge invariant operators will be induced with naturally smaller coefficients. For generic N, these plaquettes are the gauge invariant operators involving the smallest number of link fields. For $N = 2, 3$, there are gauge invariant operators involving fewer link fields (the Wilson lines wrapping cycles of the torus). These however break different chiral symmetries and are only generated at high loop order. Just as the breaking of chiral symmetries associated with the gauge couplings generates other interactions, the plaquettes induce all non-derivative gauge invariant operators. For instance, consider a closed loop ℓ which is the boundary of a region tiled by some collection of plaquettes. Then trW_ℓ is generated with a coefficient proportional to $16\pi^2 f^4$ times the product $\lambda/16\pi^2$ for all the tiling plaquettes. The natural size for a general gauge invariant operator may be worked out by keeping track of which chiral symmetries the operator breaks just as we did with the gauge couplings, remembering that the plaquette operator $tr\,{}_a\Box$ breaks the chiral symmetries associated with all the corners of ${}_a\Box$. Because of the reduced gauge symmetry at $\mathbf{0}$, we may include extra gauge invariant plaquette operators in the action. There are four such operators since only four plaquettes border the "special" site:

$$\mathcal{L}_{\mathrm{pl}_8} = f^4 tr\sqrt{12}\mathbf{T_8}\left({}_{\mathbf{0}}\Box\,\alpha_{++} + \Box_{\mathbf{0}}\,\alpha_{+-} + \Box^{\mathbf{0}}\,\alpha_{--} + {}^{\mathbf{0}}\Box\,\alpha_{-+}\right) + \text{h.c.}\,. \tag{1.9}$$

We call these special operators "T_8 plaquettes."

Finally, the standard model fermions are charged only under $\mathrm{SU}(3)_{\mathrm{color}} \times G_{\mathbf{0}}$, with their familiar standard model quantum numbers. We will see that the non-linear sigma model fields Higgs the full gauge symmetry down to the standard model $\mathrm{SU}(3)_{\mathrm{color}} \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ symmetry.

1.2 Discrete symmetries

The site, link and plaquette structure of theory space allows for discrete symmetries associated with geometric transformations. These include rotations R by 90° about an axis through the site $\mathbf{0}$ perpendicular to the theory space. This takes a site $a = (m, n)$ into $\mathbb{R}a = (-n, m)$. On a link $l = (a, b)\mathbb{R}$ acts as $R_l = (\mathbb{R}a, \mathbb{R}b)$. We also have theory space "parities" $P_{u,v}$ which correspond to reflection about the $\mathbf{u}$ and $\mathbf{v}$ axes, respectively: $P_u a = (-m, n)$, $P_v a = (m, -n)$, and similar action on the links. Note that these eight discrete transformations, comprising the point symmetry group D_4, can be generated by the two transformations R and P_u. On fields

$$RA^\mu_a = A^\mu_{\mathbb{R}a}\,, \qquad P_u A^\mu_a = A^\mu_{P_u a}\,, \tag{1.10a}$$

$$R\Sigma_l = \Sigma_{Rl}\,, \qquad P_u\Sigma_l = \Sigma_{P_u l}\,. \tag{1.10b}$$

The action of these transformations on plaquettes follows from these rules:

$$R\,{}_a\Box = \Box_{Ra}\,, \tag{1.11a}$$

$$P_u\,{}_a\Box = (\,\Box_{P_u a})^\dagger\,. \tag{1.11b}$$

If we demand that these transformations are symmetries of the action, the gauge and plaquette couplings must satisfy the relations

$$g_{\mathbb{R}a} = g_a\,, \qquad g_{P_u a} = g_a\,, \tag{1.12a}$$

$$\lambda_{\mathbb{R}a-\mathbf{u}} = \lambda_a\,, \qquad \lambda_{P_u a-\mathbf{u}} = \lambda^*_a\,. \tag{1.12b}$$

The rotational symmetry forces the four special plaquette couplings to be equal $\alpha_{++} = \alpha_{+-} = \alpha_{-+} = \alpha_{++} \equiv \alpha + i\epsilon$, while the parity symmetry forces $\epsilon = 0$. We are not obliged to impose these symmetries exactly, and in fact, our final lagrangian will preserve only the $\mathbb{Z}_4$ subgroup generated by the 90° rotation R. Other patterns of symmetry breaking are also possible.

1.3 Little Higgs

We can now discuss the origin of natural electroweak symmetry breaking in this class of models. We will later analyze the specific case $N = 2$ in detail.

Since the theory (so far) has a D_4 symmetry, we take $\epsilon = 0$, keeping only the largest couplings g_a, λ_a, and α. We will assume that these couplings are all perturbative; as we have discussed all other operators have smaller effects and may be treated as further perturbations.

In this gauged non-linear sigma model only some subgroup of the gauge symmetry is realized at low energies — many of the π_l are "eaten", giving masses to many of the gauge bosons. We will choose the plaquette couplings such that the vacuum field configuration is near $\Sigma_l = 1$. Under the uniform gauge transformation $g_a = g_{\mathrm{SU}(2)\times\mathrm{U}(1)}$ independent of a, the configuration $\Sigma_l = 1$ is invariant, showing that the diagonal $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge group is left unbroken. Thus there are massless $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ gauge bosons together with a spectrum of gauge bosons with mass scale set by gf with g a typical gauge coupling. In order to see this explicitly and identify the spectrum of the theory, we expand the lagrangian to quadratic order in A^μ_a and π_l. It is convenient to define link fields pointing in the $\mathbf{u}, \mathbf{v}$ directions as

$$\Sigma_{(a,a+\mathbf{u})} \equiv e^{i\pi^u_a}\,, \qquad \Sigma_{(a,a+\mathbf{v})} \equiv e^{i\pi^v_a}\,. \tag{1.13}$$

The lagrangian to quadratic order is

$$\begin{aligned}\mathcal{L}^{\mathrm{quad}} &= -\sum_a \frac{1}{2g_a^2} tr F_a^2 + \sum_a tr\frac{f^2}{4}\left(\partial^\mu \pi^u_a + A^\mu_a - A^\mu_{a+\mathbf{u}}\right)^2 + u \to v + \\ &\quad + \sum_a f^4 \lambda_a \,\mathrm{tr}\left(\pi^u_a + \pi^v_{a+\mathbf{v}} - \pi^u_{a+\mathbf{v}} - \pi^v_a\right)^2 + \mathcal{L}^{\mathrm{quad}}_{\mathrm{pl}_8}\,,\end{aligned} \tag{1.14}$$

where to save space we have not written the T_8 plaquette contribution explicitly. From the quadratic piece involving the gauge bosons we see explicitly that only the uniform configurations with $A^\mu_a = A^i_{\mathrm{SU}(2)}\mathbf{T}^i + A_{\mathrm{U}(1)}\mathbf{T_8}$ are massless, with all remaining $N^2 - 1$ gauge boson octets acquiring mass. Since we have $2N^2$ link scalars π_l, this means that $2N^2 - (N^2-1) = N^2+1$ scalars remain uneaten. The plaquettes give a mass to N^2-1 independent combinations of these scalars (there are N^2 plaquette terms but the sum of all N^2 linear combinations that appear vanishes, so only N^2-1 independent combinations become massive). Therefore we expect $(N^2+1)-(N^2-1) = 2$ scalars to be massless at this order. These zero modes are easy to identify in this case: they are the uniform configurations for the fields π^u_a, π^v_a

$$\pi^u_a = \frac{2u}{Nf}\,, \qquad \pi^v_a = \frac{2v}{Nf}\,, \tag{1.15}$$

where we have included the factor $2/(Nf)$ to canonically normalize u and v.

Since there is an unbroken discrete $\mathbb{Z}_4$ symmetry under which $u \to v$ and $v \to -u$, it is convenient to group u, v into a field that transforms homogeneously under this $\mathbb{Z}_4$. Defining $\mathcal{H} = (u+iv)/\sqrt{2}$, we have $\mathcal{H} \to -i\mathcal{H}$, $\mathcal{H}^\dagger \to i\mathcal{H}^\dagger$ under the $\mathbb{Z}_4$ symmetry. The components of the 3×3 matrix $\mathcal{H}$ are

$$\mathcal{H} = \frac{u+iv}{\sqrt{2}} \equiv \begin{pmatrix} \phi + \eta/2\sqrt{3} & \frac{h_1}{\sqrt{2}} \\ \frac{h_2^\dagger}{\sqrt{2}} & \frac{-\eta}{\sqrt{3}} \end{pmatrix}, \tag{1.16}$$

where under the standard model $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge symmetry, ϕ, η are complex fields transforming, respectively, in the $\mathbf{3}_0, \mathbf{1}_0$ representation, and h_1, h_2 have the quantum numbers $\mathbf{2}_{1/2}$ of the standard model Higgs.

Going beyond quadratic order these zero modes have gauge interactions under the unbroken $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge symmetry, as well as a quartic coupling which can be found by expanding the plaquette action and integrating out the heavy modes. In the case where all the λ_a are equal, in accordance with the extra-dimensional intuition, there is no heavy-light-light scalar coupling thus we do not need to integrate out the heavy modes and we find:

$$\text{constant} + 4\lambda\,\mathrm{tr}\left|\left[\mathcal{H},\mathcal{H}^\dagger\right]\right|^2 + \cdots \supset \lambda\,\mathrm{tr}\left(h_1 h_1^\dagger - h_2 h_2^\dagger\right)^2 + \lambda\left(h_1^\dagger h_1 - h_2^\dagger h_2\right)^2, \tag{1.17}$$

where $\lambda = 4/N^2\,\mathrm{Re}\,\lambda$.[1] We have again neglected the T_8 plaquettes, which do not affect the qualitative discussion. Note that the Higgs interactions are very similar to the quartic potential for the Higgses of the supersymmetric standard model (MSSM). The commutator form naturally arises from the plaquettes and is essential for obtaining a tree-level quartic potential but no tree-level mass. This is crucial for generating a natural hierarchy while incorporating hard quartic couplings. It is this feature that distinguishes these theory space models from previous attempts at obtaining the Higgs as a pseudo-Goldstone boson.

We found that, classically, we have two massless modes with quartic interactions. Radiative corrections will produce masses for these scalars, but as we have argued they are free of quadratic divergences at one-loop. Thus they are naturally light compared to the cutoff of the theory. We will call these naturally light scalars little Higgses. For a general theory space the spectrum of little Higgses can be determined from the topology of the theory space, as described in [8]. We may introduce masses for the little Higgses classically by turning on a non-zero value for ϵ. As mentioned in the beginning of the section, up to now the theory has a D_4 symmetry (under which (h_1, h_2) transform as a doublet) which enforces $\epsilon = 0$. The fermion couplings will break this D_4 symmetry, allowing us to preserve at most the $\mathbb{Z}_4$ subgroup. Although not essential, this $\mathbb{Z}_4$ symmetry has several phenomenological virtues, including helping prevent η from acquiring a vev, and we will keep it as an exact symmetry of our model.

1.4 Fermions

The standard model fermions have their conventional charges under the $G_{\mathbf{0}} = \mathrm{SU}(2)\times\mathrm{U}(1)$ symmetry; they therefore have the same charges under the diagonal subgroup that becomes the standard model $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$. In order for these fermions to acquire mass, they need Yukawa couplings to the Higgs fields, which must arise from gauge-invariant couplings to the Σ_l's. Since the Σ_l are 3×3 matrices and naturally act on triplets, we write the SM fermions as 3 components column vectors. In terms of left handed Weyl fields:

$$Q = \begin{pmatrix} q \\ 0 \end{pmatrix}, \quad U^c = \begin{pmatrix} 0 \\ 0 \\ u^c \end{pmatrix}, \quad D^c = \begin{pmatrix} 0 \\ 0 \\ d^c \end{pmatrix}, \quad L = \begin{pmatrix} l \\ 0 \end{pmatrix}, \quad E^c = \begin{pmatrix} 0 \\ 0 \\ e^c \end{pmatrix}. \tag{1.18}$$

Yukawa couplings come from gauge-invariant operators of the form

$$Q^T W_{\mathbf{0}\to\mathbf{0}} U^c\,, \tag{1.19}$$

where $W_{\mathbf{0}\to\mathbf{0}}$ is some linear combination of Wilson loops that start and end at $\mathbf{0}$. A convenient set of such Wilson loops is provided by $\mathcal{U} = \Sigma_{\mathbf{0},\mathbf{u}}\Sigma_{\mathbf{u},2\mathbf{u}}\cdots\Sigma_{-\mathbf{u},\mathbf{0}}$ and $\mathcal{V} =$

[1] For general λ_a, the potential is given by $16(\sum_a(\mathrm{Re}\,\lambda_a)^{-1})^{-1}\,\mathrm{tr}|[\mathcal{H},\mathcal{H}^\dagger]|^2$.

$\Sigma_{\mathbf{0},\mathbf{v}}\Sigma_{\mathbf{v},2\mathbf{v}}\cdots\Sigma_{-\mathbf{v},\mathbf{0}}$, together with $\mathcal{U}^\dagger$ and $\mathcal{V}^\dagger$. Under the $\mathbb{Z}_4$ discrete symmetry, $\mathcal{U}\to\mathcal{V},\mathcal{V}\to\mathcal{U}^\dagger$. It is convenient to form linear combinations that transform homogeneously under the $\mathbb{Z}_4$:

$$\mathcal{U}+\mathcal{U}^\dagger\pm\left(\mathcal{V}+\mathcal{V}^\dagger\right)\longrightarrow\pm\left[\mathcal{U}+\mathcal{U}^\dagger\pm\left(\mathcal{V}+\mathcal{V}^\dagger\right)\right], \tag{1.20a}$$

$$\mathcal{U}-\mathcal{U}^\dagger\pm i(\mathcal{V}-\mathcal{V}^\dagger)\longrightarrow\mp i\left[\mathcal{U}-\mathcal{U}^\dagger\pm i(\mathcal{V}-\mathcal{V}^\dagger)\right], \tag{1.20b}$$

where expanding to low order in u and v

$$\mathcal{U}+\mathcal{U}^\dagger\pm\left(\mathcal{V}+\mathcal{V}^\dagger\right)\sim\text{constant}+\mathcal{O}\left(\frac{\mathcal{H}^2}{f^2}\right)+\cdots,$$

$$\mathcal{U}-\mathcal{U}^\dagger+i\left(\mathcal{V}-\mathcal{V}^\dagger\right)\sim\frac{i\mathcal{H}}{f}+\cdots,\qquad \mathcal{U}-\mathcal{U}^\dagger-i\left(\mathcal{V}-\mathcal{V}^\dagger\right)\sim\frac{i\mathcal{H}^\dagger}{f}+\cdots.$$

Since the first combination begins at quadratic order in the Higgs fields, it does not produce a Yukawa coupling for the SM fermions. We therefore use the second combination in the Yukawa coupling:

$$\lambda_u f Q^T\left[\mathcal{U}-\mathcal{U}^\dagger\pm i\left(\mathcal{V}-\mathcal{V}^\dagger\right)\right]U^c, \tag{1.21}$$

where the $\mathbb{Z}_4$ is preserved with q,u^c transforming so that $(qu^c)\to\pm i(qu^c)$. Choosing the plus sign and expanding to linear order in the Higgs field, we get:

$$\lambda_u q h_1 u^c. \tag{1.22}$$

In some models, like the one we study later in more detail, only h_1 gets a vev. In that case, in order to give a non-vanishing mass to both the up and down type quarks, we choose the opposite sign for the down sector:

$$\lambda_d f D^{cT}\left[\mathcal{U}-\mathcal{U}^\dagger-i\left(\mathcal{V}-\mathcal{V}^\dagger\right)\right]Q=\lambda_d h_1^\dagger q d^c+\cdots. \tag{1.23}$$

With this choice, q,d^c transforming so that $(qd^c)\to -i(qd^c)$ preserves the $\mathbb{Z}_4$ symmetry. We can write a similar operator for the leptons:

$$\lambda_l f E^T\left[\mathcal{U}-\mathcal{U}^\dagger-i\left(\mathcal{V}-\mathcal{V}^\dagger\right)\right]L=\lambda_l h_1^\dagger l e+\cdots. \tag{1.24}$$

Each of these operators is non-local in theory space and each breaks *all* the chiral symmetries protecting the little Higgs mass. Their inclusion directly in the lagrangian would re-introduce quadratic divergences at one-loop of order $\lambda^2\Lambda^2/(4\pi)^2$. This is in contrast to the gauge couplings and plaquettes which only collectively break the chiral symmetries, introducing quadratic divergences only at higher order. The divergences generated by these non-local operators are negligible for all fields except the top quark. We will therefore generate the top Yukawa coupling from local interactions in theory space by effectively spreading the top quark out in theory space. We do this by introducing massive vector-like fermions on the sites and coupling them to the chiral fermions on the special site through link fields. Upon integrating out the massive fermions, the top Yukawa coupling is generated. The $\mathbb{Z}_4$ symmetry requires the introduction of two sets of vector-like fermions, χ's and $\tilde{\chi}$'s. We place them along the $\mathbf{u}$ and $\mathbf{v}$ axes: $\chi_{p\mathbf{u}}, \tilde{\chi}_{p\mathbf{u}}$ at the site $p\mathbf{u}$, where p is a an integer, and $\chi_{p\mathbf{v}}, \tilde{\chi}_{p\mathbf{v}}$ at the site $p\mathbf{v}$, all of which are triplets under the SU(3) gauge symmetry at the corresponding site. In addition, each of these fermions are triplets under SU(3) color. Finally they also carry charge 1/3 under the U(1) gauge factor in $G_{\mathbf{0}}$. We also introduce the conjugate of each of these fields:

$$\begin{aligned} \chi_{p\mathbf{u}}, \tilde{\chi}_{p\mathbf{u}} &\sim (\mathbf{3}_c \times \bar{\mathbf{3}})_{+\frac{1}{3}}\,, \qquad & \chi^c_{p\mathbf{u}}, \tilde{\chi}^c_{p\mathbf{u}} &\sim (\bar{\mathbf{3}}_c \times \mathbf{3})_{-\frac{1}{3}}\,, \\ \chi_{p\mathbf{v}}, \tilde{\chi}_{p\mathbf{v}} &\sim (\mathbf{3}_c \times \bar{\mathbf{3}})_{+\frac{1}{3}}\,, \qquad & \chi^c_{p\mathbf{v}}, \tilde{\chi}^c_{p\mathbf{v}} &\sim (\bar{\mathbf{3}}_c \times \mathbf{3})_{-\frac{1}{3}}\,. \end{aligned} \tag{1.25}$$

The interactions can be represented by the "traffic pattern" of figure 2. Note that the $\mathbb{Z}_4$ symmetry is manifest in this figure and will be preserved with the following charge assignment for the fermions:

$$\begin{aligned} Q &\longrightarrow e^{i\pi/4} Q \longrightarrow iQ \longrightarrow e^{i3\pi/4} Q \longrightarrow -Q\,, \\ U^c &\longrightarrow e^{i\pi/4} U^c \longrightarrow iU^c \longrightarrow e^{i3\pi/4} U^c \longrightarrow -U^c\,, \\ \chi_{p\mathbf{u}} &\longrightarrow \chi_{p\mathbf{v}} \longrightarrow \tilde{\chi}_{-p\mathbf{u}} \longrightarrow \tilde{\chi}_{-p\mathbf{v}} \longrightarrow -\chi_{p\mathbf{u}}\,, \\ \chi^c_{p\mathbf{u}} &\longrightarrow \chi^c_{p\mathbf{v}} \longrightarrow \tilde{\chi}^c_{-p\mathbf{u}} \longrightarrow \tilde{\chi}^c_{-p\mathbf{v}} \longrightarrow -\chi^c_{p\mathbf{u}}\,. \end{aligned} \tag{1.26}$$

The explicit lagrangian is given by:

$$\begin{aligned} \mathcal{L}_{\rm top} = & \; M'_L Q_3^T \left(\Sigma_{\mathbf{0},\mathbf{u}} \chi^c_{\mathbf{u}} + i \Sigma_{\mathbf{0},-\mathbf{u}} \tilde{\chi}^c_{-\mathbf{u}}\right) + M'_R \left(\chi_{-\mathbf{u}} \Sigma_{-\mathbf{u},\mathbf{0}} + i \tilde{\chi}_{\mathbf{u}} \Sigma_{\mathbf{u},\mathbf{0}}\right) U^c_3 + \\ & + e^{i\pi/4} \Big[M'_L Q_3^T \left(\Sigma_{\mathbf{0},\mathbf{v}} \chi^c_{\mathbf{v}} + i \Sigma_{\mathbf{0},-\mathbf{v}} \tilde{\chi}^c_{-\mathbf{v}}\right) + M'_R \left(\chi_{-\mathbf{v}} \Sigma_{-\mathbf{v},\mathbf{0}} + i \tilde{\chi}_{\mathbf{v}} \Sigma_{\mathbf{v},\mathbf{0}}\right) U^c_3 \Big] + \\ & + \sum_{p \neq 0} \Big\{ \chi_{p\mathbf{u}} \left(M \chi^c_{p\mathbf{u}} + M' \Sigma_{p\mathbf{u},(p+1)\mathbf{u}} \chi^c_{(p+1)\mathbf{u}} \right) + \\ & \qquad + \tilde{\chi}_{-p\mathbf{u}} \left(M \tilde{\chi}^c_{-p\mathbf{u}} + M' \Sigma_{-p\mathbf{u},-(p+1)\mathbf{u}} \tilde{\chi}^c_{-(p+1)\mathbf{u}} \right) + \\ & \qquad + e^{i\pi/4} \Big[\chi_{p\mathbf{v}} \left(M \chi^c_{p\mathbf{v}} + M' \Sigma_{p\mathbf{v},(p+1)\mathbf{v}} \chi^c_{(p+1)\mathbf{v}} \right) + \\ & \qquad\qquad + \tilde{\chi}_{-p\mathbf{v}} \left(M \tilde{\chi}^c_{-p\mathbf{v}} + M' \Sigma_{-p\mathbf{v},-(p+1)\mathbf{v}}\, \tilde{\chi}^c_{-(p+1)\mathbf{v}} \right) \Big] \Big\}\,, \end{aligned} \tag{1.27}$$

where, for notational simplicity we have chosen the same M for every site. Other realistic models with a smaller fermion structure can be found in [8]. Integrating out the massive fermions gives the top quark Yukawa coupling. Extracting this Yukawa coupling requires the diagonalization of the full mass matrix, but if all the M's are comparable, it is given parametrically by $\lambda_t \sim M/f$. This method of implementing Yukawa couplings preserves a $\mathrm{U}(2)^5$ flavor symmetry broken only by the the Yukawa matrices of the first two generations, ensuring the absence of dangerous flavor changing neutral currents.

The lagrangian

$$\mathcal{L} = \mathcal{L}_K + \mathcal{L}_{\rm pl} + \mathcal{L}_{\rm pl_8} + \mathcal{L}_{\rm fermions} + \mathcal{L}_{\rm other} \tag{1.28}$$

specifies our model, where $\mathcal{L}_{\rm other}$ includes higher dimension operators suppressed by powers of $4\pi f$ as well as non-derivative operators with naturally sized coefficients. The parameters are $g_a, \lambda_a, \alpha, \epsilon$, and the dimensionful parameters f, M's, M'_L and M'_R. g_a, λ_a and α are independently renormalized, and we take them to be perturbative with sizes $\lambda_a, \alpha \sim g_a^2$.

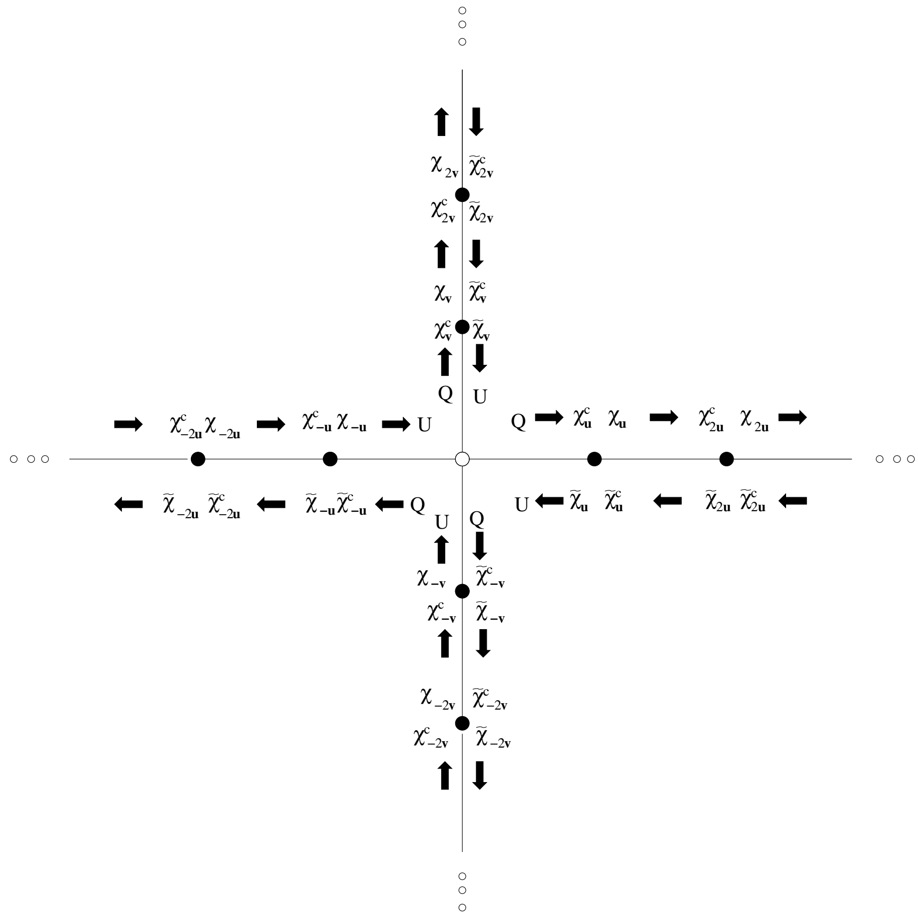

Figure 2: Representation of top quark lagrangian.

The fermion masses $M's$,M'_L and M'_R are also independently renormalized and protected by chiral symmetries. Finally ϵ, which breaks the D_4 symmetry as well as the chiral symmetries of the T_8 plaquette is then renormalized through a combination of $M'_{L,R}$ and α at two loops, giving a tiny (and irrelevant) natural size $\epsilon \sim \alpha {M'_L}^2 {M'_R}^2 / f^2 (16\pi^2)^3$.

1.5 Radiative corrections

As we have argued, the chiral symmetries protecting the little Higgs are only broken by combinations of several couplings, and therefore quadratic divergences are absent at one loop. In the case $N \geq 3$ the little Higgs potential is dominated by finite IR effects and is calculable. In the $N = 2$ case, there is in general a log divergence at one-loop and the UV and IR contributions are comparable. While our spurious symmetry analysis guarantees this result, it is reassuring to see it arise in perturbative calculation by computing the one-loop Coleman-Weinberg potential. We turn on the little Higgs fields v, and calculate the mass matrix $M(v)$ of the theory in presence of these background fields. The quadratic divergence is proportional to $trM(v)^\dagger M(v)$ and the logarithmic divergence to $\mathrm{tr}(M(v)^\dagger M(v))^2$. As an example, we look at the simple circular theory space studied in [1] with only 3 sites.

Turning on a uniform background value for the link fields: $U_i = e^{iv\sigma_3/f\sqrt{3}}$, the mass matrix squared, $M^\dagger M$, for the charged gauge bosons is proportional to:

$$\begin{pmatrix} 2 & -e^{-iv/f\sqrt{3}} & -e^{iv/f\sqrt{3}} \\ -e^{iv/f\sqrt{3}} & 2 & -e^{-iv/f\sqrt{3}} \\ -e^{-iv/f\sqrt{3}} & -e^{iv/f\sqrt{3}} & 2 \end{pmatrix} . \tag{1.29}$$

We can easily see that $tr M^\dagger M$ does not depend on v since v does not appear anywhere on the diagonal. In terms of mass eigenstates, turning on v increases the mass squared of light fields but decreases the mass of heavy fields. It is similarly easy to see that $\mathrm{tr}(M^\dagger M)^2$ does not depend on v, so there is neither quadratic nor log divergence at one loop in this model. In the $N = 2$ case, the mass matrix is proportional to:

$$\begin{pmatrix} 2 & 2\cos(\frac{v}{f\sqrt{2}}) \\ 2\cos(\frac{v}{f\sqrt{2}}) & 2 \end{pmatrix} . \tag{1.30}$$

Again there is no v on the diagonal and thus no quadratic divergence. However $\mathrm{tr}(M^\dagger M)^2 = 8(1 + \cos^2(v/f\sqrt{N}))$, and so there is a log divergence in this case.

In our model with a 2 dimensional lattice of sites the mass matrices are larger, but the same observations hold true. Radiative corrections generate mass terms:

$$V_{\text{quadratic}} = m_1^2 |h_1|^2 + m_2^2 |h_2|^2 + m_\phi^2 \,\mathrm{tr}|\phi|^2 + m_\eta^2 |\eta|^2 . \tag{1.31}$$

Note that there is no $h_1^\dagger h_2$ term or linear term in η due to the $\mathbb{Z}_4$ symmetry. If $m_\phi^2, m_\eta^2, m_2^2 > 0$ and $m_1^2 < 0$, we will trigger electroweak symmetry breaking (EWSB). Since the quartic potential vanishes for $h_1 = h_2$, we also require $m_1^2 + m_2^2 > 0$ to avoid rolling away along this flat direction. One might worry that after EWSB, ϕ and η could get large vevs. However, the $\mathbb{Z}_4$ symmetry which leads to an unbroken $\mathbb{Z}_2$ symmetry even after electroweak symmetry breaking forbids odd powers of ϕ and η. With positive $m_{\phi,\eta}^2$, both vevs vanish.

For $N \geq 3$, the mass terms in (1.31) are dominated by finite contributions which are in principle calculable. In the $N = 2$ case, most of them are log divergent at one loop and therefore not calculable. However, a spurion analysis shows that even in that case, the mass *difference* between h_1 and h_2 is dominated by IR effects and is finite at one loop. In the limit where Yukawa couplings are neglected, the theory has a D_4 symmetry under which h_1 and h_2 form a doublet and are therefore degenerate. Consequently, the mass difference must come from fermion loops. By computing the fermionic contribution to the one-loop Coleman-Weinberg potential when $N = 2$, we find that this is indeed the case, the result being:

$$-\left(\frac{3M^2 {M'_R}^2 {M'_L}^2}{2\pi^2 f^2 \left({M'_R}^2 - {M'_L}^2 \right)} \log \frac{M^2 + 4{M'_R}^2}{M^2 + 4{M'_L}^2} \right) |h_1|^2 + \mathcal{O}\left(h^4\right) . \tag{1.32}$$

This is negative for any values of M'_L, M'_R and therefore can trigger EWSB in a natural way. This is an improvement over [1] where EWSB was triggered by a non-zero ϵ; here it is driven by top quark radiative corrections, much as in the MSSM.

1.6 Scales

In the very low energy theory, keeping only the classically massless modes, the Higgs would get a quadratically divergent mass. It is the appearance of the heavy modes that cancel this divergence. For example it is the appearance of heavy "KK" gauge bosons at energy gf/N which cancels the divergence of the low energy gauge boson loop. Similarly, it is the heavy fermions introduced to "delocalize" the top quark which cancel the divergent top quark loop of the very low energy theory. Finally, the heavy scalars which get masses from the plaquettes are necessary to cancel the divergence from the Higgs quartic couplings. This observation allows us to estimate the various scales of the theory such as the Higgs mass and the cutoff $4\pi f$. For example the one loop gauge contribution to the Higgs mass is approximately

$$m_h^2 \sim \frac{g_{LE}^2}{16\pi^2}\left(\frac{gf}{N}\right)^2 \sim \left(\frac{g_{LE}^2}{16\pi^2}\right)^2 (4\pi f)^2 \sim \left(\frac{\alpha_{LE}}{4\pi}\right)^2 \Lambda^2\,, \tag{1.33}$$

where $g_{LE}^2 = g^2/N^2$ is the low energy coupling, i.e. the coupling of the unbroken gauge group. This is to be contrasted with the standard model case where:

$$m_h^2 \sim \left(\frac{\alpha_{LE}}{4\pi}\right) \Lambda^2\,. \tag{1.34}$$

The presence of an extra $\alpha_{LE}/4\pi$ factor in our case allows us to take the cutoff of our theory parametrically above the TeV scale, near $\sim$ 10–100 TeV, and still have perturbative new physics at the TeV scale stabilizing the Higgs. A similar conclusion can be reached for the quartic couplings.

The same simple estimate can be done for the top quark contribution, but it is more model dependent:

$$m_h^2 \sim \frac{\lambda_t^2}{16\pi^2} M^2 \sim \left(\frac{\lambda_t^2}{16\pi^2}\right)^2 (4\pi f)^2 \sim \left(\frac{\alpha_t}{4\pi}\right)^2 \Lambda^2\,, \tag{1.35}$$

where λ_t is the top Yukawa coupling given parametrically by $\sim M/f$. Again the additional $\alpha_t/4\pi$ factor allows us to choose the cutoff in the ~ 10 TeV region.

To summarize, in the very low energy theory beneath the masses of the heavy modes, we have the standard model with 2 Higgs doublets and other scalars in the ~ 100 GeV region. At roughly 1 TeV, heavy fermions, scalars and vector bosons responsible for stabilizing the Higgs mass appear. Finally, at $\sim$ 10–100 TeV our description of the physics in terms of non-linear sigma model fields breaks down and a UV completion is needed. Although we can imagine many different possibilities for this UV completion (such as SUSY, new strong dynamics, low scale quantum gravity, etc.), they do not affect our description of electroweak symmetry breaking and we do not discuss it here. The important point is that the cutoff of our theory (where it becomes strongly coupled and a UV completion is needed) is parametrically above the scale where perturbative new physics appears to stabilize the weak hierarchy.

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1.7 Dark matter

As we have seen, our theory has a natural geometric $\mathbb{Z}_4$ symmetry, generated by the 90° rotation R on theory space. The scalars ϕ, η, h_1, h_2, together with the standard model fermions, are charged under this $\mathbb{Z}_4$, and it is then broken along with the electroweak symmetry when the Higgs acquires its vev. However the combination P of R^2 with a gauge transformation Ω and a phase transformation on the fermions remains unbroken even after electroweak breaking:

$$P = R^2 \Omega e^{i\frac{\pi}{2}F}\,, \qquad \Omega = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{1.36}$$

Where F is a fermion number under which Q, L, χ's and $\tilde{\chi}$'s have charge $+1$ and U^c, D^c, E^c, χ^c's and $\tilde{\chi^c}$'s have charge -1. Under P, the standard model Higgses and fermions are even,

$$h_{1,2} \longrightarrow h_{1,2}\,, \qquad \psi_{SM} \longrightarrow \psi_{SM} \tag{1.37}$$

while ϕ, η are odd

$$\phi \longrightarrow -\phi\,, \qquad \eta \longrightarrow -\eta\,. \tag{1.38}$$

Since this is an unbroken symmetry even after EWSB, ϕ and η do not acquire vevs as claimed earlier. Furthermore, the lightest scalar in the ϕ, η sector will be exactly stable. Depending on the parameters of the theory, this stable particle may be neutral, a linear combination of ϕ_3 and η. This particle with a mass in the $\sim 100\,\text{GeV}$ range provides a natural WIMP candidate for the dark matter of the universe, with weak-scale mass and cross-section.

2. $N = 2$

In this section we examine the smallest model, with $N = 2$, in detail. The price we pay for using such a small moose is the presence of small loops in the diagram, which are associated with operators generated with log divergent coefficients at one-loop. These operators must then be included in our lagrangian with arbitrary coefficients of natural size $\sim g^4/(4\pi)^2$. For $N \geq 3$, the natural size of the analogous operators is much smaller, and therefore negligible compared to the finite one-loop radiative correction.

The gauge group of this model is $\mathrm{SU}(3)^3 \times \mathrm{SU}(2) \times \mathrm{U}(1)$. The $\mathbb{Z}_4$ symmetry allows for 2 independent SU(3) couplings, but for simplicity we will take them to have a common value g_3. The SU(2) and U(1) gauge couplings are g_2 and g_1. The moose diagram for the model is shown in figure 3. The relevant parts of the lagrangian are the kinetic terms, the 4 plaquettes, the 4 T_8 plaquettes, the fermion interactions and the operators generated with log divergence mentioned earlier. For this particular model, the notation introduced earlier for the general N torus is incomplete since there are two different links between every pair of adjacent sites. Therefore, instead of writing down formulæ, we will represent these operators by pictures. The operators that can be generated purely through gauge interactions are represented in figure 4. We call the resulting operators $\mathcal{O}$, $\mathcal{O}'$, $\mathcal{O}_8$ and $\mathcal{O}'_8$.

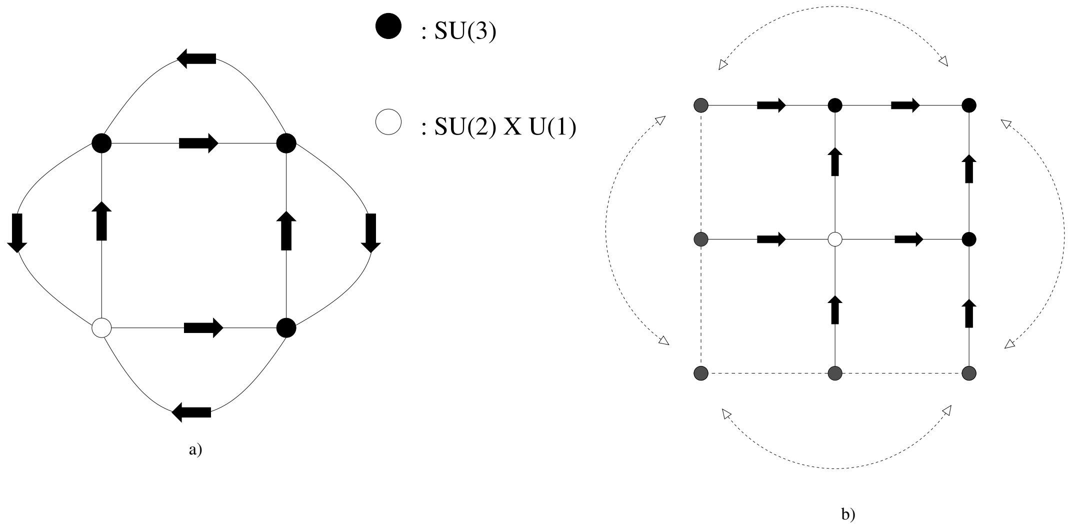

Figure 3: Moose diagrams for the $N = 2$ torus. b) is an unfolding of a). There are 4 independent sites: in b), any two sites related by a reflection through the vertical or the horizontal axis are identified. An example of this identification is shown by the dotted arrows. There are 8 independent link fields represented by the thick arrows. a) shows clearly the sites and links while b) makes the plaquettes and all the discrete symmetries apparent.

These operators appear with coefficient of natural size $g^4/(4\pi)^2 f^4$. We also have two operators induced through plaquette interactions, $\mathcal{O}_2$ and $\mathcal{O}_2^8$, shown in figure 5. These operators need two plaquettes to be generated and have natural size $\sim \lambda^2/(16\pi^2) f^4 \sim g^4/(4\pi)^2 f^4$. Thus our lagrangian contains:

$$\mathcal{L}_{\rm other} = \frac{g_3^4}{16\pi^2} f^4 \left(a\mathcal{O}_1 + a'\mathcal{O}_1' + a_8 \mathcal{O}_1^8 + a_8' \mathcal{O}_8' + b\mathcal{O}_2 + b_8 \mathcal{O}_2^8 \right) + \cdots , \tag{2.1}$$

where the ellipses represent subdominant terms of order $g^6/(4\pi)^4$ and higher. Note that we have scaled the coefficients of these operators with the largest gauge coupling; their natural sizes might somewhat smaller, corresponding to small values of the a's and b's.

2.1 Very low energy theory

The very low energy theory contains the SM particles, two Higgs doublets h_1, h_2, one complex SU(2) triplet scalar ϕ and one complex singlet scalar η. The scalar potential comes from three sources: first the plaquettes give a quartic interaction for the little Higgses. Second, the operators in $\mathcal{L}_{\rm other}$ give mass terms. We assume that these mass squared terms are all positive. Finally, in order to break the electroweak symmetry, we need a negative mass squared for the Higgs. As advertised, this is achieved through the one loop top Yukawa contribution to the Higgs potential which was given in eq. (1.32):

$$-\left(\frac{M^2 {M_R'}^2 {M_L'}^2}{2\pi^2 f^2 \left({M_R'}^2 - {M_L'}^2 \right)} \log \frac{M^2 + 4{M_R'}^2}{M^2 + 4{M_L'}^2} \right) |h_1|^2 + \mathcal{O}\left(h^4\right) \equiv -\Delta_{m_1}^2 |h_1|^2 . \tag{2.2}$$

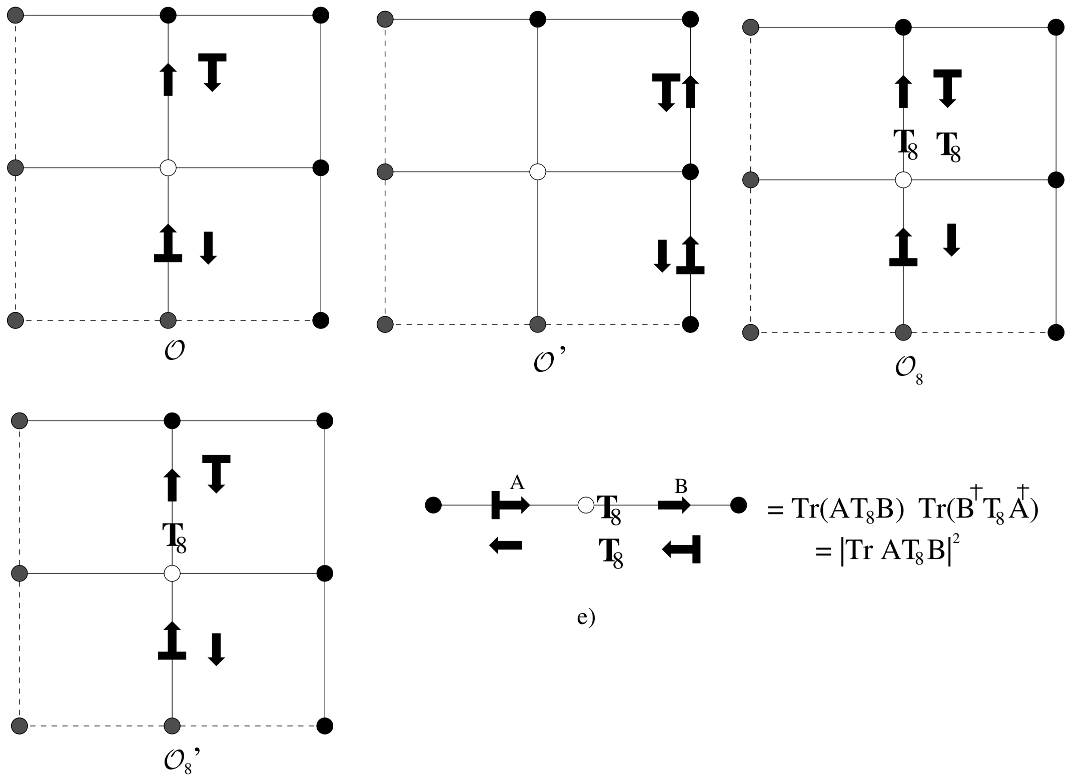

Figure 4: Operators $\mathcal{O}$, $\mathcal{O}'$,$\mathcal{O}_8$ and $\mathcal{O}'_8$ included in the $N = 2$ torus lagrangian. They consist of a product of links forming a closed path. The links included in the product are represented by the arrows, and the large base indicates a trace should be included before the corresponding link. The T_8 indicates that a T_8 matrix should be inserted in the product. The final $\mathbb{Z}_4$ invariant operators are obtained by summing over the 90° rotations of these pictures. e) is an example of this notation relevant for $\mathcal{O}_8$.

To quartic order, the potential for the Higgs doublets is then given by:

$$V(h_1,h_2) = 2\left(\lambda - \frac{\alpha}{2}\right)\left(|h_1|^4 + |h_2|^4\right) - 2(\lambda+\alpha)\left|h_1^\dagger h_2\right|^2 - \\ - 2(\lambda - 2\alpha)|h_1|^2|h_2|^2 + \left(m^2 - \Delta^2_{m_1}\right)|h_1|^2 + m^2|h_2|^2\,, \tag{2.3}$$

where m^2 is given in term of the coefficients appearing in $\mathcal{L}_{\rm other}$:

$$m^2 = \frac{g_3^4 f^2}{16\pi^2}\left[-24(a + a' + b) + 6(b_8 + a'_8)\right]. \tag{2.4}$$

Assuming $m^2 < \Delta^2_{m_1}$ and $2m^2 - \Delta^2_{m_1} > 0$, h_1 gets a vev:

$$h_1 = \begin{pmatrix} 0 \\ \frac{v}{2} \end{pmatrix}, \qquad v^2 = \frac{\Delta^2_{m_1} - m^2}{2\left(\lambda - \frac{\alpha}{2}\right)} = (246\,\mathrm{GeV})^2\,. \tag{2.5}$$

The physical spectrum is easily derived in unitary gauge where:

$$h_1 = \begin{pmatrix} 0 \\ \frac{(v+h_0)}{\sqrt{2}} \end{pmatrix}, \qquad h_2 = \begin{pmatrix} H^+ \\ \frac{(H_0 + iA_0)}{\sqrt{2}} \end{pmatrix}. \tag{2.6}$$

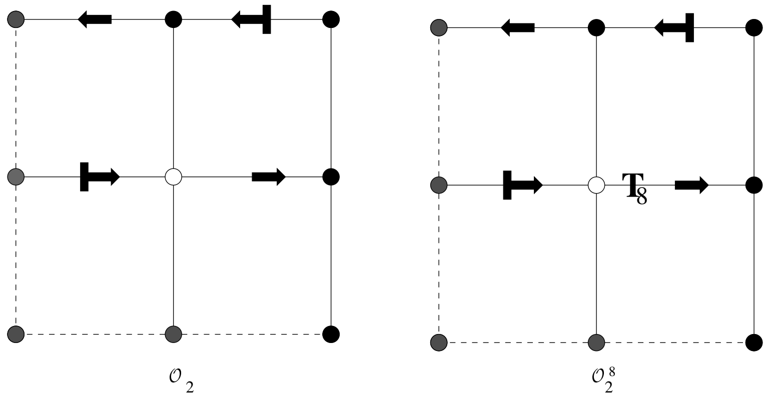

Figure 5: Operators $\mathcal{O}_2$and $\mathcal{O}_2^8$ can be generated by the plaquette. Again, they are gauge invariant products of the links represented by arrows, the large base representing a trace and they are implicitly symmetrized by 90° rotations.

The masses are:

$$\begin{aligned} m_{h_0}^2 &= 2\left(\Delta_{m_1}^2 - m^2\right) = 4\left(\lambda - \frac{\alpha}{2}\right) v^2 \,, \\ m_{H_0}^2 &= m_{A_0}^2 = 2m^2 - \Delta_{m_1}^2 \,, \\ m_{H^+}^2 &= m^2 - (\lambda - 2\alpha)\, v^2 . \end{aligned} \tag{2.7}$$

Note that in this model H_0 cannot decay to fermions or gauge bosons at tree level. After EWSB, the quartic potential generates cubic terms of the form $H_0\phi\phi, H_0\phi\eta, H_0\eta\eta$ and therefore these decays will dominate if kinematically allowed. Otherwise H_0 will decay at loop level, for instance to gauge bosons via light scalar loops. Similar considerations apply to H^+. Next we turn to the triplet and singlet masses. There are two electrically charged scalars, ϕ_1^+ and ϕ_2^+, with masses

$$\begin{aligned} m_{\phi_1^+}^2 &= -(\lambda + \alpha)\, v^2 + m^2 - 18\frac{g_3^4}{16\pi^2}(b_8 + a_8')f^2 \,, \\ m_{\phi_2^+}^2 &= \left(2\lambda + \frac{\alpha}{2}\right) v^2 + m^2 - 18\frac{g_3^4}{16\pi^2}(b_8 + a_8')f^2 \,. \end{aligned} \tag{2.8}$$

The mass splitting can be related to the Higgs doublets masses:

$$m_{\phi_1^+}^2 - m_{\phi_2^+}^2 = -\frac{1}{4}m_{h_0}^2 + 2\left(m_{H_0}^2 - m_{H^+}^2\right). \tag{2.9}$$

There are also 2 complex neutral scalars, N_1 and N_2, which are linear combinations of ϕ_3 and η. The lightest of ϕ_1^+, ϕ_2^+, N_1 and N_2 is stable since all these are odd under the discrete P symmetry introduced in section 1.7. Phenomenologically, we need the neutral states to be lightest for dark matter. The charged states will then typically decay to neutral states along with (on or off-shell) W's.

2.2 Heavy modes

In addition to the little Higgses, the theory contains heavy TeV scale modes that are responsible for stabilizing the weak scale: 3 SU(2) triplets, $W^a_{1,2,3}$, 3 SU(2) doublets $V^i_{1,2,3}$ and 3 SU(2) singlet $B_{1,2,3}$ vector bosons ; 1 complex and 1 real SU(2) triplet ϕ'_1, ϕ'_3, 3 SU(2) doublets $h'_{1,2,3}$ and 1 complex and 1 real SU(2) singlet scalars η'_1, η'_3. There are also heavy fermions which we denote collectively by χ. Table 1 gives the masses of these modes before EWSB, their quantum numbers, their main decay modes and their transformation properties under the unbroken P symmetry introduced in section 1.7. For gauge bosons, the masses are calculated by diagonalizing the mass matrix found by setting the link fields to unity in the kinetic term. The scalar mass matrix is extracted from the plaquette potential, expanding the exponentials to quadratic order. Likewise, the interaction between the different modes are given by expanding the plaquettes and kinetic terms. The kinetic terms give vector-vector-scalar and scalar-scalar-vector couplings while the plaquettes give scalar-scalar-scalar coupling. It is straightforward to see that, when the little Higgses are uniform in theory space, there are no little Higgs-little Higgs-heavy scalar couplings.

2.3 Spectrum

As an illustration, we have chosen a typical set of parameters and calculated the full spectrum of the theory. For the input parameters in table 2, the resulting spectrum is shown in figure 6. With this choice of parameters, the lightest Higgs mass is 174 GeV at tree level, and the UV completion scale is at 10 TeV. It is interesting to compare the values of the input coefficients for the special operators with the low-energy one-loop log-divergent radiative corrections, ignoring the log factor. In all cases, these input values are comparable to the one-loop corrections, demonstrating the absence of fine-tuning in these couplings. Finally, the finite negative contribution to the Higgs mass squared from the large top Yukawa coupling is $-(460\,\mathrm{GeV})^2$. For our light Higgs mass $\sim 174\,\mathrm{GeV}$, this required a bare positive mass squared of $+(442\,\mathrm{GeV})^2$. These mass squared choices are moderately tuned at the 10% level. This modest tuning can be reduced either with a somewhat heavier Higgs or with lighter masses for the vector-like χ fermions.

3. Conclusion

We have studied a class of realistic models of electroweak symmetry breaking in which the Higgs appears as an extended object in theory space. This class of models, first introduced in [1], is based on toroidal theory spaces. We reviewed and extended the analysis of the $N \times N$ torus and studied the $N = 2$ case in detail.

Although our analysis was based on a specific theory space, the mechanism of EWSB applies more generally.[2] A robust prediction of all such models is the presence of at least two naturally light Higgs doublets. This is a consequence of the Higgs quartic couplings

[2]The general rules for building such models, together with the procedure for identifying the little Higgses from topological properties of the theory space, will be discussed in [8].

particle	spin	mass^2	SU(2)	U(1)	P	decay
W_1^a	1	$g_3^2 f^2 \beta_2^-$	**3**	0	$+$	$f^+ f^-$
W_2^a	1	$g_3^2 f^2$	**3**	0	$+$	$\phi\phi^\ddagger$
W_3^a	1	$g_3^2 f^2 \beta_2^+$	**3**	0	$+$	$f^+ f^-$
B_1	1	$g_3^2 f^2 \beta_1^-$	**1**	0	$+$	$f^+ f^-$
B_2	1	$g_3^2 f^2$	**1**	0	$+$	$hh^\ddagger$
B_3	1	$g_3^2 f^2 \beta_1^+$	**1**	0	$+$	$f^+ f^-$
V_1^i	1	$g_3^2 f^2(1-\sqrt{2}/2)$	**2**	1/2	$-$	$\phi^\dagger h$
V_2^i	1	$g_3^2 f^2$	**2**	1/2	$-$	$\phi h^\ddagger$
V_3^i	1	$g_3^2 f^2(1+\sqrt{2}/2)$	**2**	1/2	$-$	$\phi^\dagger h$
h_1'	0	$16(\lambda - \frac{\alpha}{2})f^2$	**2**	1/2	$+$	$t\bar{t}, t\bar{b}$
h_2'	0	$16(\lambda - \frac{\alpha}{2})f^2$	**2**	1/2	$+$	$h_1^\dagger \phi_3', h_1^\dagger \eta_3', h_1^\dagger W_2, \phi^\dagger V_2, h_2^\dagger W_{1,3}, t\chi_1^c$
ϕ_1'	0	$16(\lambda + \alpha)f^2$	**3**	0	$-$	${h_3'}^\dagger h_1, \phi W_2, \phi^\dagger W_{1,3}, t\chi_{5,}^c$
η_1'	0	$16(\lambda - \alpha)f^2$	**1**	0	$-$	$h_2 V_2, h_2^\dagger V_{1,3}, t\chi_5^c$
h_3'	0	$32(\lambda - \frac{\alpha}{2})f^2$	**2**	1/2	$-$	$h_2^\dagger \phi_1', WV_2^\ddagger, t\chi_3^c$
ϕ_3'	0	$32(\lambda + \alpha)f^2$	**3**	0	$+$	${h'}_1^\dagger h_2, t\chi_4^c, WW_2^\ddagger$
η_3'	0	$32(\lambda - \alpha)f^2$	**1**	0	$+$	$h_1^\dagger h_2', t\chi_4^c$
χ_1	1/2	M^2	**1**	$\pm 2/3$	$+$	$h_2 t, h_2 b$
χ_2	1/2	M^2	**1**	$\pm 2/3$	$-$	ηt^c
χ_3	1/2	M^2	**1**	$\pm 2/3$	$-$	ηt^c
χ_4	1/2	M^2	**2**	$\pm 1/6$	$+$	$h_2 t^c$
χ_5	1/2	M^2	**2**	$\pm 1/6$	$-$	$\phi t, \eta t, \phi b, \eta b$
χ_6	1/2	M^2	**2**	$\pm 1/6$	$-$	$\phi^\dagger t, \eta^\dagger t, \phi^\dagger b, \eta^\dagger b$
χ_7	1/2	$M^2 + 4{M_L'}^2$	**2**	$\pm 1/6$	$+$	$h_1 t^c$
χ_8	1/2	$M^2 + 4{M_R'}^2$	**1**	$\pm 2/3$	$+$	$h_1 t, h_1 b$

Table 1: Summary of heavy modes. Here $\beta_i^\pm = 1/2(2 + g_i/g_3 \pm \sqrt{2 - 2g_i^2/g_3^2 + g_i^4/g_3^4})$. The decay mode with the ‡ symbol vanish when all the f's are equal.

descending from plaquette interactions, producing commutator squared potentials. In the specific models studied in this paper there are additional weak triplets and singlets scalar at low energy. Theory spaces often have geometric discrete symmetries, such as the $\mathbb{Z}_4$ symmetry which is broken to a $\mathbb{Z}_2$ after electroweak symmetry breaking in the model we discussed. The light triplets and singlets are odd under this symmetry and therefore the lightest of these states is stable, providing a WIMP dark matter candidate. We also showed that the top quark can drive the Higgs mass negative and trigger EWSB in a natural way. At TeV energies, new physics appears to stabilize the weak scale. This new physics takes the form of new heavy particles which soften each of the quadratic divergences of the low energy theory: heavy gauge bosons cancel quadratically divergent gauge boson loops, heavy scalars cancel the quadratic divergence from the low energy quartic couplings and heavy fermions cancel the quadratic divergence from the top loop.

parameter	input value	1 loop vector	1 loop scalars
Λ	$10\,\mathrm{TeV}$		
f	$795\,\mathrm{GeV}$		
g_1	0.21		
g_2	0.995		
g_3	1.5		
M	$950\,\mathrm{GeV}$		
M'_R	$1100\,\mathrm{GeV}$		
λ	0.18		
a	1.06		
a'	0.3		
b	-1.8		
a'_8	-0.08	-0.034	0
a_8	0.04	-0.0052	0
b_8	-0.07	0	$0.135y$
m^2	$(442\,\mathrm{GeV})^2$	$(226\,\mathrm{GeV})^2$	$(202\,\mathrm{GeV})^2$

Table 2: Input parameter used to generate the spectrum. M'_L is then fixed by the measured top Yukawa coupling to be $M'_L = 1060\,\mathrm{GeV}$, and α is fixed by the scale of EWSB $v = 246\,\mathrm{GeV}$ to be $\alpha = 0.1$. Note that m^2 is given as a function of the other parameters (see eq. (2.4)).

In the $N = 2$ case, we computed the full spectrum of the theory for a typical set of parameters, and briefly discussed the primary decay modes for the new fields, leaving a more detailed study of both the collider phenomenology and cosmological implications of this theory to future work.

How do our theories compare with supersymmetric models of electroweak symmetry breaking? One of the features that makes supersymmetry attractive is its ability to accommodate precision electroweak data, as a consequence of its weakly coupled description of EWSB. Since our models are weakly coupled at the TeV scale, they share this success. In supersymmetry, there is often a flavor problem associated with flavor-changing neutral currents for the light generations, mediated by superpartners. In our models, the only new states which carry flavor are the heavy fermions which "delocalize" the top quark in theory space, and so there are no flavor-changing interactions involving the first two generations at the TeV scale. (This is similar to the situation in the "more minimal" supersymmetric models with heavy first two generation scalars [9, 10]).

The supersymmetric standard model also has the virtue that it can be extrapolated to energies much higher than the TeV scale. A remarkable success of such an extrapolation is the supersymmetric prediction of gauge coupling unification. This prediction remains one of the most compelling arguments for supersymmetry and the energy desert.

By contrast, our effective theory description of the physics breaks down at relatively low scales, $\sim$ 10–100 TeV. There are straightforward UV completions of our non-linear sigma models into linear sigma models, but like the standard model, these themselves suffer from

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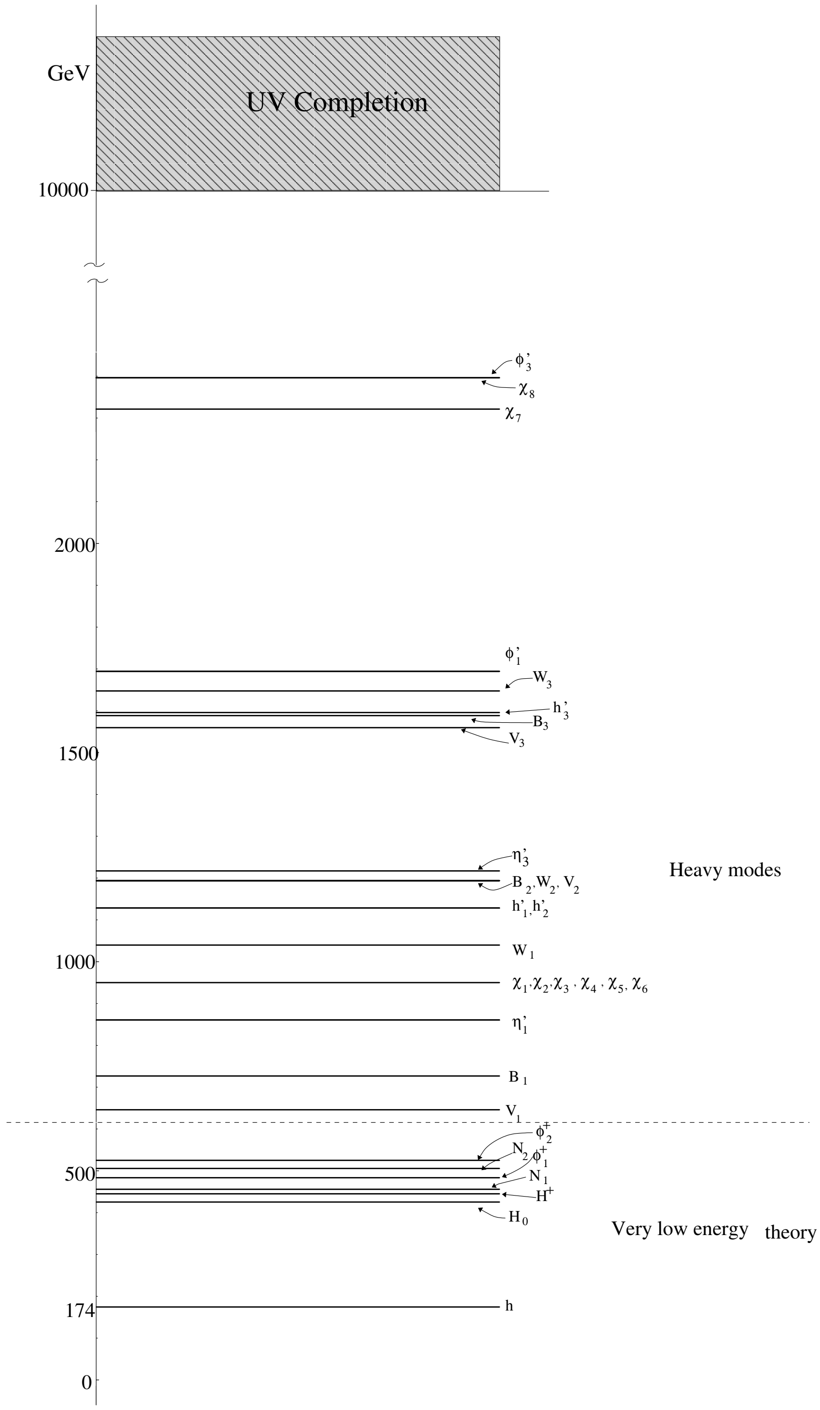

Figure 6: Spectrum of the theory.

a (slightly reduced) "hierarchy problem". We can instead consider supersymmetrizations of these linear sigma models above $\sim$ 10–100 TeV, or perhaps strongly coupled gauge dynamics or even low scale gravity models above this scale. Whatever this UV completion physics is, it completely decouples from the physics of EWSB, and is therefore irrelevant for upcoming collider experiments. As for gauge coupling unification, there have been a number of recent proposals on retaining this success at low energies (see for instance [11, 12]), or recovering the correct prediction of the weak mixing angle from different group-theoretic structures [13, 14]. It would be interesting to pursue combining these ideas with our models, and some work along these lines is already in progress [15].

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